Clusters of Objects and Associations between Variables: Two Approaches to Archaeological Classification

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PART I

Introduction

It is useful to keep in mind a clear distinction between two topics related to classification. One topic concerns the archaeological tasks and the questions, either social-scientific or culture-historical, for which classification, grouping, or clustering procedures may be relevant. In this context, the tasks and the research questions are primary, and one asks whether classification procedures are indeed the techniques of choice for performing the tasks and answering the questions. If they are, then which approaches to classification are the most useful and efficient? If not, then in what ways do “typological” approaches impede our understanding?

The second topic involves pure (if not very sophisticated) mathematics, and considers only the formal properties of possible classification or grouping procedures. These formal properties can be explored without, for the moment, concerning oneself with the relevance of the procedures to any task or question of real anthropological interest. The point of such inquiry, at least for an archaeologist, is, of course, that the results will have a bearing on the first topic. Nevertheless, it is useful to spend some time examining formal aspects of various tech-
niques, without being unduly concerned with arguments about archaeological–anthropological theory. This chapter is concerned mainly with this second topic, although considerations of archaeological purpose do occasionally manifest themselves.

My aim here is to resolve, or at least elucidate, recent controversy between F. R. Hodson and A. C. Spaulding. Briefly, Hodson (Doran and Hodson 1975, Hodson, Chapter 2 of this volume) holds that seeking associations between attributes is quite different from object clustering, and that object-clustering techniques are generally much preferable and are closer to common archaeological practices and to well-founded archaeological intuitions. Spaulding (1977) argues, on the other hand, that the approaches are closely related, and that there are very good reasons for frequently using attribute-association approaches.

First, some terms and definitions. I use the term *assemblage* to refer to a specific set of material objects. It should go without saying that everything we do with such a set presupposes that the set is *relevant* and *appropriate* for some important archaeological problem or purpose. Here, however, I focus on the formal aspects of analysis, and simply assume that we are applying our techniques to worthwhile sets of objects and variables. Following Spaulding (1977) I use the term *variable* to refer to a particular kind of observation on an object, and *attribute* to refer to a particular value or range of values of a variable. Thus *red* is an attribute of the variable *color*, and 6.5 to 7.0 *cm* is an attribute of the variable *length*. *State*, *value*, and *score* are synonyms of *attribute*.

The formal problem, then, can be put in terms of a set (or assemblage) of *N* discrete objects, each of which is described by one and only one value for each of *M* variables. A further stipulation here is that space–time coordinates, including provenience, are either excluded from the variables under consideration or, if they are included, are simply treated on an equal footing with variables such as color, material, length, and weight.

The aim, then, is to *discover* and *succinctly express* whatever there is which is not random, nor chaotic, nor monotonously uniform, in the rectangular *N* × *M* matrix of objects and variables.

**Object Clustering**

The object-clustering approach starts with the familiar notion of grouping similar objects together, and putting different objects into different groups. It may or may not be possible to do this in some unproblematic way. In order for a technique of this type to work very well, we have to be able to partition the assemblage into subsets which,
as Hodson (Doran and Hodson 1975:159) puts it, exhibit both "internal cohesion and external isolation." To make this idea more precise, we have to define some function of the descriptive variables as a measure of similarity or dissimilarity. This measure has to constitute a scale that is at least sufficiently ordered so that, for any three objects in the assemblage, A, B, and C, we can always say that A is more similar to B than to C, less similar to B than to C, or equally similar to both B and C. Our intuitive notion of an ideally well-behaved type, then, is given by Newcomer and Hodson as "a discrete group of related tools where each tool is more similar to all other tools of its type than to any tool of another type" (1973:90–91, as quoted by Spaulding 1977:3, italics added). Figure 3.1 illustrates, in a simplified two-variable space, a set of objects that can easily and unambiguously be divided into five clusters, each of which perfectly conforms to this concept of type.

In practice, real data rarely behave this well, and to insist that clusters must fully fit such a rigid definition in order to be usefully typelike at all would be like insisting that a correlation, to be useful, has to be exactly unity. A preferable definition of an object-cluster type is a subset of an assemblage, each member of which is more similar to at least one other member of the same subset than to any member of the assemblage that is not in the same subset, and where at least most members of the subset are more similar to at least most other members of the same subset than to at least most members of the assemblage.

**FIGURE 3.1.** An assemblage which subdivides into five "perfect" types by object-clustering. The horizontal and vertical axes represent the scales on two variables used to describe the objects. Points represent individual objects.
that are assigned (by the same criteria) to any other type. This is a
disconcertingly intricate piece of verbiage, but the concepts are quite
simple. The definition is not intended to specify a unique classification
algorithm. Instead, it captures what I think the essence of the object-
cluster type concept is, while remaining ambiguous enough to include
a variety of classification procedures as special cases. It requires that
no object should be more similar to some object assigned to another
type than to any other object assigned to the same type, but allows
that some objects assigned to different types may be more similar to one
another than each is to some of the objects in the types to which each
is assigned. For example, in Figure 3.2, object A is assigned to Type
III, while object B is assigned to Type II, despite the fact that A is more
different from C (also in Type III) than from B. The basis for assigning
A to Type III is that A is considerably more similar to D (as well as
several other objects) than to B or any other object not in Type III. We
are not unduly bothered by the fact that A is less different from B (and
a few other items in Type II) than it is from C (and a few other items
in Type III).

It is also important to note that it may be impossible to partition an
assemblage into disjoint typelike subsets that are also exhaustive. There
may be some unclassifiable leftovers that may be "transitional." That
is, their similarities to both of two types are so evenly balanced that
assignment to either type seems highly arbitrary. In Figure 3.2, object
F is transitional between Types III and V. Other objects may be un-
classifiable because they are "deviant" and very dissimilar to all other
objects in the assemblage. In Figure 3.2, object E is a deviant that has extreme values on both variables and is not very similar to any of the five types.

Data sets such as that illustrated in Figure 3.2 do not allow very good types to be obtained by object clustering. However, they are common in practice. Subsets that satisfy the less stringent criteria I stated earlier can still be said to have a useful degree of "typehood." We can, and in practice do, tolerate a moderate proportion of transitional and deviant objects. However, if the proportion is very high, the object-clustering approach breaks down. In particular, if there are too many transitional objects the second of the two requirements which Hodson stresses is violated; "external isolation" of types is lost. Also, if clusters are too long and straggly (as is often the case when "single-linkage" hierarchical clustering algorithms are used), then "internal cohesion" is absent. Just as we can characterize subsets of an assemblage by their degree of typehood, we can characterize the whole assemblage by its degree of "typability." An assemblage is perfectly typable if (as in Figure 3.1) it can be broken down into disjoint subsets where the stringent Newcomer and Hodson ("each . . . all . . . any") criteria fully apply.

Perfect untypability would occur if, as is logically possible, there is no way to partition the assemblage into more than one or fewer than $N$ disjoint nonempty subsets that avoid a high proportion of subset members being just about as similar to many nonmembers as to most objects that are members of the same subset. For example, imagine a tiny assemblage with only four objects: A, B, C, and D. Suppose A is equally similar to both B and D but less similar to C; B is as similar to A as to C but less similar to D; C is as similar to B as to D but less similar to A; and D is as similar to C as to A but less similar to B. Figure 3.3 represents this situation. There is no way of dividing such an assemblage into good item-cluster types except by assigning each

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**FIGURE 3.3.** An assemblage consisting of four objects. Arrows point from each object to the other two objects which are most similar to it. All these maximum similarity scores are ties.
object to a unique type. Table 3.1 is the matrix of similarity coefficients between all possible pairs of the four objects.

Most real assemblages will be neither perfectly typable nor perfectly untypable, but will represent some degree of typability, as in Figure 3.2. For such assemblages, a useful way to state Hodson's stress on both internal cohesion and external isolation for a good object-cluster type is to say that, for any member of a good type, the frequency distribution of similarities to all other objects in the assemblage should be bimodal or polymodal. There will be a relatively large number of high similarities (for comparisons with other objects belonging to the same cluster), a distinctly lower frequency of intermediate-level similarities (mostly for transitional objects), and one or several additional frequency peaks (for objects in other clusters and deviant objects). Figure 3.4 illustrates the general form of the frequency distribution of similarity coefficient values between an object that belongs to a good type and all other objects in the assemblage.

For an object that belongs to a perfect type, it would be possible to specify a point on the horizontal axis of Figure 3.4 such that all higher similarity values pertain to comparisons with objects belonging to the same type as the reference object, whereas all lower similarity values pertain to objects that are not in the same type. This point will be close to point X in Figure 3.4. If the reference object belongs to a less-than-perfect type, it will still be true that there is a point, not far from X, such that most of the higher similarities pertain to comparisons with objects in the same type, and most of the lower similarities pertain to objects not in the same type. I should note that since every object in an assemblage can be taken as the reference object, there are potentially as many frequency distributions (similar to Figure 3.4) as there are objects in the assemblage. The degree to which there is a well-defined dip in the frequency distribution, like that at point X in Figure 3.4, for most objects in a type, is an indicator of how well the type fits the "external isolation" criterion.

<table>
<thead>
<tr>
<th>Table 3.1</th>
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<tbody>
<tr>
<td>Similarity Coefficients between All Possible Pairs of the Four Objects in Figure 3.3*</td>
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<tr>
<td></td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

*Zeros indicate low similarity, ones indicate high similarity, and twos indicate identity.
FIGURE 3.4. The general pattern of the frequency distribution of similarity coefficients between an object which belongs to a good object-clustering type and all other objects in the assemblage. Most objects whose similarity is higher than the value marked X belong to the same type as the reference object. Most objects with lower similarity belong to other types or are unassigned.

Associations between Variables

Spaulding (1977) argues that the approach which I call variable association is usually preferable to object clustering. In this approach we wish to detect any way in which the state or value of one variable which an object exhibits is relevant for the states of other variables exhibited by the object. At least roughly, two kinds of relevance can be described. If knowledge of the state of variable X for an object is directly relevant for stating the probabilities of the various possible states of variable Y for the same object, we say that variables X and Y are associated. If knowledge of the state of variable Z for an object is relevant for saying how the states of X relate to the states of Y, we say that there is an interaction between Z and X and Y. For the remainder of this chapter I ignore the distinction between association and interaction, and will use association as a generic term implying that the values of one variable (or set of variables) are somehow relevant for the values of some other variable (or set of variables).
According to Spaulding, it is this search for patterning or relationship between variables that is fundamental. However, one may find that certain combinations of values of different variables occur much oftener than would be expected if the frequencies of the combinations of variable values were simply what one would expect from randomly combining the different values of the different variables. If so, then the objects that share such an unexpectedly frequent combination of values can usefully be labeled a type, and the combination itself can be called a value cluster, or, more commonly, an attribute cluster. Thus, investigation of associations between variables can form the basis for a classification technique, although Spaulding (1977) now regards the classification as a useful byproduct of the investigation, rather than its main purpose.

To digress briefly about the terms attribute cluster and variable association, the former stems in part from the tendency in some fields to use attribute in the sense that, following Spaulding, I use variable here. My (Cowgill 1977) use of attribute as more or less a synonym for variable (pointed out by Hodson 1977) was an unfortunate lapse that I should have avoided. However, attribute cluster can also refer to the particular combinations of states (values, or, in their proper sense, attributes) that, because of their exceptional frequency, are used to define a specific type. Because of the lack of uniformity in the definition of attribute this is probably a poor word to use, and it would be better to speak of value combinations or value clusters and variable associations. Variables interact or are associated if and only if the frequencies of certain combinations of values of the variables are higher than (and necessarily, the frequencies of certain other value combinations are lower than) would be predicted from the univariate frequencies of the values of each single variable. A value combination that is unexpectedly common is, then, a value cluster.

**PART II**

Hodson argues that one important advantage of the object-clustering approach is that it enables one to make the similarity coefficient a function of very heterogeneous types of variables, including both discontinuous and continuous. For reasons which I hope will become clear, I think this is not a very happy way of characterizing different types of scales, and I much prefer the terms nominal, ordinal, and interval or ratio, although I recognize that many other types and subtypes of scales can be defined.
Based on the discussion in Part I, three questions can now be posed:

1. Given an assemblage described (for whatever reason) entirely in terms of nominal scale variables, what relation holds between object-clustering and variable-association approaches to analysis? More specifically, do they generate equivalent types, radically different types, or something in between?

2. Given an assemblage whose description includes at least one ordinal- or interval-scale variable, what relation holds between object-clustering and variable-association approaches?

3. For what purposes, and under what circumstances, is it advisable to transform ordinal or interval scale variables into nominal variables before proceeding with further analysis?

**Assemblages Described Wholly in Terms of Nominal Variables**

We can obviously always label as a type every distinct value combination that is actually exhibited by at least one object in the assemblage. In a sense, this satisfies the "internal-cohesion-external-isolation" criterion of Hodson, since every state or value of a nominal variable is simply different from every other one, and there is no such thing as an intermediate value. This means, for example, that if we define a similarity coefficient on the basis of just one variable, the coefficient can have only two values: same and different. Necessarily, the frequency distribution of similarities is bimodal. Furthermore, classification on the basis of values of a single nominal variable is often highly relevant to sensible problems (flint versus obsidian, male versus female, etc.) and it is quite common in archaeological practice. In its own terms, there can be no objection to this procedure, as far as it goes.

The point is, of course, that it does not go very far in telling us what is going on in our assemblage. It simply amounts to listing all the logically possible values or value combinations that actually occur, and calling each a type. I think we feel happiest (with good reason) about classification on a single nominal variable when the relevance of the variable is obvious, as is the case for obsidian versus flint. But suppose we have an assemblage of pottery, some of which is incised with spirals and others with squiggles. I think that if we could see no way whatever in which spiral-decorated and squiggle-decorated pots differed, or differed systematically, except for the spirals or the squiggles, we would feel that the difference between spirals and squiggles was not very important, and it would be a passionate splitter indeed who would
establish two types on the basis of such a minor difference. I think that the way in which an object clusterer would approach this problem would be to ask: "Do the pots with squiggles in any other way, or in general, tend to be more like other pots with squiggles than like pots with spirals?"

But this, of course, leads us directly to considerations of two or more variables. Consider the simplest case: two dichotomous variables. Suppose we have an assemblage of pots described in terms of two variables: shape (with values bowl and jar) and temper (with values sand and shell). In terms of this impoverished set of variables, the assemblage can be fully described by a familiar $2 \times 2$ contingency table, whose cells give the frequencies of shell-tempered bowls, sand-tempered jars, etc. We can, of course, consider each of the four distinct logically possible shape–temper combinations (or at least each actually to occur) a type, or we could group any two or three (or, trivially, all four) of the combinations in any way which we deem useful for some specific purpose. But I would prefer to reserve the more noncommittal term *category* for subdivisions of the assemblage based on sets of value combinations which are determined by considerations *other than* (or additional to) internal relations among the $N$ objects and $M$ variables of our assemblage description. I think that the notion underlying both object-clustering and variable-association approaches to classification is that types represent something discovered about the relations between objects in an assemblage, considered wholly in terms of the variables chosen for description of the assemblage. In theory we may base our selection of the variables and their possible values entirely on considerations of their relevance for specific purposes, but in practice—both in intuitive classifications and in many formal approaches (e.g., Christenson and Read 1977)—the tendency seems to be to begin with a sizable number of *possibly relevant* variables and to decide that the truly relevant ones are those variables that in fact do, in terms of their patterning within the assemblage, show some sort of structure.

For our simple example, the procedure specified by the variable-association approach is clear. From the total frequency of shell-tempered vessels, the total frequency of sand-tempered vessels, of bowls, and of jars, we compute the number of sand-tempered jars, sand-tempered bowls, etc., which would be expected if jars were just as likely as bowls to be sand-tempered, shell-tempered vessels were just as likely to be jars as bowls, and so on. In short, we assume that propensity to possess a specific tempering material has nothing to do with propensity to possess a specific shape. In terms of formal analysis, incidentally, I speak as if it were the objects themselves which manifested propen-
sities. As archaeologists, we should of course be concerned with the propensities of the people who made and used the objects (whether or not those propensities derived from conscious norms, "mental templates," or the like). However, the data actually amenable to formal analysis consist of objects and the propensities of their properties to relate to one another. Care, technical skill, and ingenuity in obtaining appropriate assemblages are necessary but not sufficient conditions for being able to relate propensities of objects to propensities of people.

The frequencies of the value combinations actually observed in the assemblage may be so close to those expected, if propensity for a given shape were in fact independent of propensity to possess a given tempering material, that the differences could easily be explained by chance. Or, even if the association is significant, there may be no good evidence for more than a very slight uninteresting relationship between the propensities. In either case, there is no good basis for subdividing the assemblage into distinct types. Consider Table 3.2. For this table, \( \phi = .0077 \), \( \chi^2 = .06 \), and there is good evidence that temper and shape are essentially independent variables. There is no association between the variables and there is no basis for terming any one (or any combination) of the four value combinations a value cluster.

Table 3.3, in contrast, illustrates another assemblage in which the number of sand-tempered jars greatly exceeds the frequency range which would be at all probable, if propensity to possess sand temper had nothing to do with shape. For Table 3.3, \( \phi = .848 \) and \( \chi^2 = 719 \). The association between variables is both strong and statistically highly significant. The value combination sand-tempered jar constitutes a pronounced value cluster. For a \( 2 \times 2 \) contingency table, it is logically necessary that, if one value combination is unexpectedly frequent, the

<table>
<thead>
<tr>
<th>Shape</th>
<th>Sand</th>
<th>Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jar</td>
<td>240</td>
<td>245</td>
</tr>
<tr>
<td></td>
<td>(238.1)</td>
<td>(246.9)</td>
</tr>
<tr>
<td>Bowl</td>
<td>251</td>
<td>264</td>
</tr>
<tr>
<td></td>
<td>(252.9)</td>
<td>(262.1)</td>
</tr>
<tr>
<td></td>
<td>491</td>
<td>509</td>
</tr>
</tbody>
</table>

*Values expected on the hypothesis of independence are in parentheses.
opposite combination must also be unexpectedly frequent, and the
remaining two combinations unexpectedly infrequent. In Table 3.3, the
value combination shell-tempered bowl is also a strong value cluster, and
I suppose the combinations sand-tempered bowl and shell-tempered jar
might be called anticlusters.

From the Spaulding, variable-association viewpoint, the analyses of
Tables 3.2 and 3.3 seem clear, uncomplicated, and sharply different.
Table 3.2 shows negligible relation between temper and shape, whereas
the assemblage described by Table 3.3 shows a very strong tendency
for sand temper to occur in jars and shell temper in bowls.

From an object-clustering viewpoint, the analysis of these two tables
is a little more complicated. As I understand Spaulding (1977) he claims
that Hodson—and Hodson may well agree with this—would say that
both the assemblage of Table 3.2 and of Table 3.3 exhibit four distinct
types; the only difference being that in Table 3.2 all four types occur
with approximately equal frequency, whereas in the assemblage of
Table 3.3 it happens that two of the four types are much less common
than are the other two types.

As long as we consider tables where all marginal frequencies are
nearly equal, as is the case with Tables 3.2 and 3.3, there may be little
to choose between the object-clustering and the variable-association
approaches. However, even here I am inclined to think that most ar-
chaeological practitioners of object sorting would see a considerable
difference between the two assemblages. In the case of Table 3.3, surely
most archaeologists would see two very good types: sand-tempered
jars and shell-tempered bowls, with a residue of atypical or transitional
objects; bowls that were sand tempered and jars that were shell tem-

<table>
<thead>
<tr>
<th>Shape</th>
<th>Sand</th>
<th>Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jar</td>
<td>450</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>(238.1)</td>
<td>(246.9)</td>
</tr>
<tr>
<td>Bowl</td>
<td>41</td>
<td>474</td>
</tr>
<tr>
<td></td>
<td>(252.9)</td>
<td>(262.1)</td>
</tr>
<tr>
<td></td>
<td>491</td>
<td>509</td>
</tr>
</tbody>
</table>

*Expected values are in parentheses.
pered. On the other hand, the treatment of the assemblage of Table 3.2 might be much more variable. Depending on assessments of importance (or weight) of shape versus temper—assessments which necessarily would have to derive from evidence or considerations external to Table 3.2—different persons might describe four types, or they might say there is a jar type (with variable temper) and a bowl type (also with variable temper) or they might define a shell-tempered type (of variable shape) and a sand-tempered type (also of variable shape). Table 3.2 does seem to show data that are, from traditional intuitive object-clustering viewpoints, more ambiguous and less satisfactory than Table 3.3.

To make the difference between Tables 3.2 and 3.3 more clear, consider the frequencies of interobject similarities. As long as no external criteria are introduced to justify giving one variable a greater weight than another, we have to begin, at least, by giving each variable equal weight. This implies, for equally weighted nominal variables, that any similarity coefficient has to be some monotonic function of the number of variables on which the two objects share identical values. For two nominal variables, equally weighted, the similarity coefficient can assume just one of three values: high (the objects have identical values on both variables), medium (the objects have identical values on exactly one variable), and low (the objects have different values for both variables).

Consider a sand-tempered jar in the assemblage of Table 3.2. Figure 3.5 shows the frequency of similarity coefficients derived from comparison with the other 999 objects of the assemblage. Figure 3.5 is analogous to Figure 3.4.

In Figure 3.5 we see that the similarity coefficient frequency distribution is polynominal only if we think of each of the three possible values as separated by a gulf from every other value. It seems to me that the distribution is polynominal only if we agree that every ordinal scale has as many modes as it has ranks. There is a very reasonable sense in which the pattern of similarity coefficients in Figure 3.5 is unimodal, and in that sense, the category sand-tempered jar of Table 3.2 shows only internal cohesion but not external isolation. Most objects in the assemblage of Table 3.2 are, in fact, transitional between sand-tempered jars and shell-tempered bowls, since they resemble sand-tempered jars on one variable and shell-tempered bowls on the other variable.

If we focused on any of the other three categories of Table 3.2, the frequency distribution of similarity coefficients would be similar to Figure 5, and would again, in the same sense, be unimodal.

The frequency distribution of similarity coefficients between a sand-tempered jar of the Table 3.3 assemblage, and all other objects in the same assemblage, is shown in Figure 3.6. Here, there is far more clearly,
FIGURE 3.5. The frequency distribution of similarity coefficients between a sand-tempered jar and all 999 other objects in the assemblage shown in Table 3.2.

FIGURE 3.6. The frequency distribution of similarity coefficients between a sand-tempered jar and all 999 other objects in the assemblage shown in Table 3.3.
by any definition, a bimodal distribution. Consider, however, the distribution of similarity coefficients for sand-tempered bowls in the assemblage of Table 3.3. Figure 3.7 shows an overwhelmingly unimodal distribution, in very sharp contrast to Figure 3.6, which represents sand-tempered jars within the same assemblage. Bowls with sand temper are not only rare, they also represent a combination usually avoided, and might be called an antitype.

In general, for $M$ equally weighted nominal variables, the similarity coefficient can have up to $M + 1$ distinct ranked values. I believe that

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**FIGURE 3.7.** The frequency distribution of similarity coefficients between a sand-tempered bowl and all 999 other objects in the assemblage shown in Table 3.3.
the internal cohesion–external isolation concept amounts to demanding that, for a good type, the observed frequencies of interobject similarities between any member of the good type and all other objects of the assemblage should show at least one pronounced minimum (surrounded by at least two maxima) comparable to Figures 3.4 and 3.6 and unlike Figures 3.5 and 3.7. Although I have not constructed a formal proof, the evidence of Tables 3.2 and 3.3 should make it perfectly apparent that, for two nominal variables, so long as marginal totals are roughly similar, the necessary and sufficient condition for this requirement to be met is that there be substantial association between variables. If we allow one or both variables to have three or more values, it seems that this remains the necessary and sufficient condition. Furthermore, I cannot see that considering three or more nominal variables changes this conclusion.

In short, it appears to me that if we (a) require a good object-clustering type based on nominal variables to show internal cohesion–external isolation at some level beyond the most direct sense in which any value of any nominal variable is discontinuously different from any other value, and if (b) the total frequencies of the observed instances of each value of a given variable are all roughly comparable, then the object-clustering and the variable-association approaches to discovering types are logically equivalent. Good object-clustering types can be discovered if and only if strong variable associations occur.

However, suppose some marginal totals are much larger than others. Table 3.4, for example, represents an assemblage in which shell-tempered vessels greatly outnumber sand-tempered, and bowls greatly outnumber jars. From the object-clustering viewpoint there are no good

<table>
<thead>
<tr>
<th>Shape</th>
<th>Sand</th>
<th>Shell</th>
<th>Marginal Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jar</td>
<td>10</td>
<td>290</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>(90.0)</td>
<td>(210.0)</td>
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</tr>
<tr>
<td>Bowl</td>
<td>290</td>
<td>410</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>(210.0)</td>
<td>(490.0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>700</td>
<td>N = 1000</td>
</tr>
</tbody>
</table>

*Sand-tempered jars are far less abundant than would be expected from the moderately low frequencies of sand temper and jar shape. Expected values are in parentheses.
types here, although one might well observe that sand-tempered jars are highly atypical. However, no matter which value combination category one focuses on, the frequency distribution of interpair similarity coefficients will be unimodal, like Figures 3.5 and 3.7, rather than bimodal like Figures 3.4 and 3.6.

For the data of Table 3.5, the object-clustering approach leads to much the same conclusion as for the Table 3.4 data. To be sure, a traditional reaction to these data might be to say that shell-tempered bowls are a good type, and that all three other value combinations are rather atypical. It is also true that, for shell-tempered bowls, the frequency distribution of similarity coefficients shows a very strong mode for high, rather than medium. But, no matter which category of Table 3.5 we focus on, there is no way of getting a bimodal distribution of similarity coefficients.

In short, Tables 3.4 and 3.5 are, from an object-clustering point of view, similar to Table 3.2 and unlike Table 3.3. Unless one labels each distinct discrete value combination a type, there are no good object-clustering types in any of these three assemblages. From the viewpoint of variable associations the situation is quite different. In Table 3.4 \( \phi = .381 \) and \( \chi^2 = 145 \). There is a statistically very significant and moderately strong association between the variables. Assuming that choice of temper had nothing to do with vessel shape, we would expect to have observed about 90 sand-tempered jars—much less than the expected frequency of any other value combination, but still far more than the actually observed 10. Thus, Table 3.4 gives evidence of a strong tendency to avoid using sand temper for jars, which is not simply a consequence of the facts that sand temper and jar shapes are moderately

<table>
<thead>
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<th>Table 3.5</th>
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<td>Hypothetical Assemblage with Very Unequal Marginal Totals*</td>
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*Sand-tempered jars are far more abundant than would be expected from the low frequencies of sand temper and jar shape. Expected values are in parentheses.
uncommon. This is an interesting finding, which cannot be stated at all well by calling sand temper plus bowl shape and shell temper plus jar shape value clusters; although in fact these are the two value combinations which occur with unexpectedly high frequency in the Table 3.4 data. It certainly goes against practice and intuition to refer to sand-tempered bowls and shell-tempered jars as types in this example.

In Table 3.5, $\chi^2$ is again highly significant ($\chi^2 = 49$) and the association is moderately strong ($\phi = .22$). Here, however, the use of sand temper in jars is strongly favored. Given the overall scarcity of both jars and sand temper, one would expect only about 10 sand-tempered jars if choice of temper had nothing to do with shape. In fact, although the category of sand-tempered jars is by far the sickest of all in this assemblage, there are three times as many sand-tempered jars as would be expected. Only 8% of the bowls are sand tempered, but 30% of the jars are sand-tempered. Evidently the persons responsible for this assemblage were much less reluctant to use sand temper in jars than in bowls. It would be of interest to attempt to explain this discovery. Similarly, for the assemblage of Table 3.4, it would be interesting to seek an explanation for our evidence that sand temper in jars was so unpopular.

My point, then, is that when marginal totals are not approximately equal, it is possible to have sizable and interesting associations between variables that are not revealed or well expressed by object-clustering approaches.

For a $2 \times 2$ table, is the converse true? That is, if marginal totals are highly unequal, can one have good object-clustering types in cases where the variable associations are extremely weak? If we define good types as those that have a bimodal distribution of similarity coefficients, the answer is a definite no. If the variable associations are negligible, then there is no way of cooking marginal totals to get a bimodal distribution of similarity coefficients.

For a $2 \times 2$ table with highly unequal marginal totals, a necessary but not sufficient condition for good object-cluster types is that there be a strong association between the variables. On the other hand, good object-cluster types are a sufficient, but not necessary, condition for strong variable associations to exist.

In sum, for pairs of nominal variables with some values much more frequent than others, the variable-association approach is distinctly preferable. Although I have not extended the analysis fully to the general case of $M$ nominal variables with three or more values apiece, it seems pretty clear that the same considerations hold. Furthermore, the new methods of discrete multivariate analysis, expounded by Spaulding
(1977 and elsewhere) offer important further opportunities for insight, and I know of nothing comparable offered by the object-clustering approach.

I should conclude this section by stressing that I believe I am much less critical of agglomerative numerical taxonomic object-clustering approaches than either Spaulding (1977) or Christenson and Read (1977) appear to be. Nevertheless, as long as one is working entirely with nominal variables, the investigation of associations between variables appears to be more powerful; in principle capable of revealing and succinctly expressing anything discoverable by an object-clustering approach and capable also of revealing patterning which the latter approach cannot.

In terms of practical archaeological applications, as opposed to formal analysis, the principal limitation of the variable-association approach with nominal variables is that, as Spaulding admits, the analysis becomes very complex if more than four or five variables are considered jointly. On the other hand, the analysis can include an unlimited number of objects without difficulty. Object-clustering procedures can handle an unlimited number of variables, but have difficulty with more than a few hundred objects (and some, such as multidimensional scaling, have trouble with more than a few score). Also, the resulting clusters are hard to interpret if more than a few variables are involved. At present it is neither feasible nor desirable to carry through either approach in a fully formal way if one wants to include a moderately large number of variables and a really sizable number of objects—the situation which occurs routinely in the archaeology of complex societies. The value of investigating various formal approaches is partly that it provides a rationale and standards which can be approximated in the less formal processing of large amounts of archaeological data. But consideration of explicit formal methods also should provide an incentive for deriving smaller data subsets which it is feasible to analyze formally, and which also can be the basis for generalizations about entire assemblages. Objects can be obtained from large assemblages by some form of probability sampling and one can go through a number of stages of preliminary analysis in search of a small set of highly relevant and highly interesting variables.

**Assemblages Whose Description Includes at Least One Interval- or Ordinal-Scale Variable**

A discussion of object clustering on the basis of a single ordinal or interval scale variable must begin with examination of the concept of
dissection. We can tabulate the frequency distribution of the scores for all the objects in the assemblage. For an interval variable or an ordinal variable with many tied scores, there may be ranges of the variable in which relatively few or no scores occur. Figure 3.2 (although it refers to a similarity coefficient rather than a data variable) shows such a gap in the region around point X. An ordinal variable with few ties could not, by its nature, exhibit any such well-defined gaps. However, I cannot think of archaeological data involving ordinal scales with few ties, so this minor complication can be ignored. If the frequency distribution of an interval or ordinal variable has two or more modes with gaps in between, if the gaps cannot easily be due to chance, and if there is good reason to think that the assemblage reflects the behavior of something approximating a single human community during a period too brief to involve much culture change, then it is difficult to explain the gaps unless the differences somehow mattered to the ancient makers and users of the objects. Thus, the existence of a multimodal frequency distribution does more than furnish a procedure for grouping objects. It is also evidence, in terms of widely accepted theory about human behavior, that the differences between the groups were somehow culturally meaningful. I stress the somehow, because I do not want to become involved in controversy about whether types based on such clusters can properly be called etic, or whether we have succeeded in "getting into the minds of the natives." In contrast, absence of clear and significant univariate multimodality fails to provide such evidence (although the negative evidence is weak, since subsequent analysis may reveal multimodality when the joint distribution of two or more variables is considered).

Object clustering, then, can proceed by using the gaps in the frequency distribution to set limits to groups. Within each group there is only one frequency mode, and groups are separated from one another by value ranges in which the scores of few or no objects occur. Thus, on the basis of a single variable, the internal cohesion and external isolation criteria can be applied.

It is, however, always possible to subdivide any set of objects on the basis of their scores on an interval or ordinal variable. A dissection is such a subdivision made when there are no well-defined modes and gaps in the frequency distribution, or a subdivision that disregards modes and gaps when they do occur. Sometimes a dissection can be useful. For example, if we have hundreds of artifacts, ranging in length from 5 to 20 cm, it will be sensible to group them into 15–30 arbitrary groups, each defined by an interval 1 or .5 cm long. Such a procedure, however, does not yield theoretically meaningful groups; it is instead
a useful step in the search for such groups. In general, dissections which lump a wide range of values into just a few broad intervals are not merely theoretically meaningless; they actually impede the discovery of any possibly meaningful groups.

Now, the significance of Hodson’s use of the term *discontinuous* to refer to nominal scale variables is that, as I understand him, he argues that *every* nominal scale variable provides discontinuities which are not arbitrary and which enable us to do object-clustering on the basis of a single variable. The implication, if I understand him correctly, is that dissection is simply not a problem for a well-chosen nominal variable. Flint is discontinuously different from obsidian, male is discontinuously different from female, and that is all there is to it.

In a sense, I have no quarrel with this. In terms of archaeological purpose, there is no question that it is extremely useful to avoid lumping the flint with the obsidian. The point is, with the categories of a single nominal variable, if there is no such thing as a dissection, there is also no such thing as a nondissection. The distinction between categories that are or are not dissections simply breaks down (unless one begins to lump distinct states of the variable, which is another matter). The point is that the discontinuities between states of a single nominal variable are not evidence about ancient behavior in the way that gaps in the distribution of an interval variable constitute evidence. The discontinuities in a nominal variable are *given* by the inherent nature of such a scale. To obtain evidence that the different states of a nominal variable mattered to the makers and users of an assemblage, we have to consider their joint distribution with the states of at least one other variable.

We can now explore the differences between object-clustering and variable-association approaches to interval variables. Consider a pair of interval-scale (more strictly, ratio scale) variables such as length and width. Figure 3.8 illustrates an assemblage of points in which length and width are very highly correlated. An object-clustering approach to this assemblage (insofar as it considered only length and width) would find no basis for partitioning the assemblage into types, and would have nothing to say about the assemblage except that it was not subdividable into types. On the other hand, a variable-association approach would establish that length and width were extremely highly correlated. This would be an empirical regularity of some interest—it is somewhat surprising that the relationship should hold so well over a wide range of lengths. Also, if we were to extend the analysis to multivariate techniques involving more than two variables, it is clear that length and width could be replaced by some single factor or dimension which
they both reflect. In short, for the assemblage represented in Figure 3.8, investigation in terms of length and width proceeds better by way of variable association than by way of object clustering.

Consider another assemblage in which the length and width of objects is distributed as in Figure 3.9. Clearly the linear correlation between length and width is practically zero. Although I hesitate to say that R-mode principal components analysis or other forms of variable-association approach would shed no light at all on the situation, it is clear that object-clustering approaches would work better. Indeed, given a set of real projectile points with lengths and widths corresponding to the points of Figure 3.9, ordinary "eyeball" human pattern recognition would probably work quite well at sorting them into two rather good types.

To make the problems with the variable-association approach in this situation more clear, consider Figure 3.10. Clearly, at least on the basis of these two variables, this assemblage can only be dissected. In contrast to the assemblage in Figure 3.9, there are no good object clusters here. Yet the univariate frequency distributions and the correlation coefficients are practically the same for each assemblage. Any approach, such as principal components or factor analysis, which operates on a matrix of correlation coefficients, will fail to detect much difference between the two assemblages. This can be read as a warning to inspect scat-
FIGURE 3.9. Scattergram of length versus width in a hypothetical assemblage where the linear correlation between length and width is nearly zero, and the least-squares regression line is a very poor summary of the data. Both univariate frequency distributions are approximately unimodal. However the bivariate distribution shows two good clusters.

FIGURE 3.10. Scattergram of length versus width in a hypothetical assemblage. The linear correlation and univariate frequency distributions are approximately the same as for the assemblage in Figure 3.9. However, in this assemblage there are no good clusters.
terplots before launching a multivariate analysis, but it also points up the fact that in some situations object clustering may be a preferable procedure.

It appears, then, that for interval scale variables, as well as for ordinal variables whenever there are many tied scores, the variable-association and the object-clustering approaches are complementary. Each is well adapted to providing kinds of information which are provided with difficulty or not at all by the other approach. In general, object-clustering approaches provide information that approximates traditional concepts of classification. Variable-association approaches, while providing important information, do not provide results which lend themselves to being expressed in terms of types.

When Is It Advisable to Transform Ordinal or Interval Variables into Nominal Variables?

Spaulding (1977:3) argues that when our object is "the search for culturally imposed order in a collection of artifacts," we should transform ordinal or interval variables into nominal variables. He argues that if, considered univariately, an interval variable shows just one good mode, then we can assume that the variety shown by the objects in our assemblage merely represents different realizations of a single ideal value for that variable. If, on the other hand, two or more good modes are observed, we are entitled to assume that each mode represents a distinct ideal value for the variable, and values clustering about each mode can and should be treated as discrete values of a nominal variable. Thus, if length, measured in millimeters, shows well-defined modes for about 35 mm, 65 mm, and 100 mm we should regard length as a nominal variable with three values: shortness, mediumness, and longness. A point that is 31 mm long is neither more nor less different from a point 64 mm long than from a point 102 mm long, on the level of culturally imposed order.

I am very uncomfortable about this view, for several reasons. First, as is illustrated here by Figure 3.9, and as is pointed out by Hodson (Chapter 2 of this volume, and in his discussion of Figure 7.2 of Doran and Hodson 1975:173), an interval variable which is unimodal when considered by itself, may be bimodal or polymodal when its joint multivariate distribution is considered. Second, I believe that real-world assemblages, whether we look at univariate frequency distributions or joint multivariate distributions, are often neither clearly unimodal nor clearly polymodal. I think that often a considerable subjective element
intervenes in deciding which modes, of a rather ragged distribution, should be taken seriously. At the very least, considerable discussion and effort must go into deciding which and how many modes are culturally meaningful in the kind of irregular frequency distributions one often gets with real assemblages. Finally, I see the search for culturally imposed order, as defined by Spaulding, as a strong commitment to emic analysis. I consider such analysis possible and important, but there are many other highly important questions for which a more etic approach is appropriate. Even if I were fully convinced that nominal variables are the only ones appropriate for emic analysis, I would still very much want to keep interval and ordinal scale information for other kinds of problems. Clay (1976) gives one example of cultural interpretations which required preservation of interval scale information. And certainly, variable-association approaches to interval and ordinal variables remain worthy of cultivation and elaboration.

I will, however, close with a conjecture. It is possible for the objects in an assemblage to show an interesting degree of polymodality when joint distributions over three or more variables are considered, even though no univariate or joint bivariate distributions are polymodal, and even though all linear correlations are quite low. So far as I know, there is no good multivariate statistical way for dealing with this situation.\(^1\) It seems to be the kind of situation in which old-fashioned intuitive pattern recognition may work better (or less badly) than anything more formal. I wonder, however, if it would be useful to explore such a situation by breaking each interval variable up into a moderately large number of nominal states, regarding this operation frankly as a dissection, in order to bring to bear techniques of discrete multivariate analysis, with the aim of possibly discovering nonlinear interactions between the variables. If there is polymodality in the joint distribution of any subset (or all of) the variables, such a technique should in

\(^1\) This is, of course, the situation that Sokal and Sneath type numerical taxonomy is supposed to deal with. Despite all the criticism of that approach—its failure to use all available information, and difficulties in telling which variables mainly account for a particular polythetic cluster—I think that if one works with due regard for its limitations, this particular kind of object-clustering approach can be quite useful. In particular, I feel that Christenson and Read (1977) do not make a fully convincing case for the superiority of a factor-analytic approach because (a) the bulk of their comparisons are with a clustering algorithm that does not very closely approximate the internal cohesion as well as external isolation concept of a good object-cluster type and (b) in comparing the more reasonable (and much more successful) average-linkage algorithm with factor-analysis results, they seem to merely assume that whenever there are discrepancies, it is because average-linkage clustering is wrong, rather than that the factor-analysis results could be wrong or at least expressing different reasonable alternatives.
principle be able to detect it. The questions are (a) whether in fact it
is the mathematically most feasible alternative for this problem, and
(b) whether, if one dissects variables into enough discrete states to be
able to capture any existing modes, the number of cells in the relevant
contingency tables will be so large that useful results will be possible
only for unrealistically large assemblages.

REFERENCES

Christenson, Andrew L., and Dwight W. Read

Clay, R. Berle

Cowgill, George L.

Doran, J. E., and F. R. Hodson

Newcomer, M. H. and F. R. Hodson

Spaulding, Albert C.